PROTON - PROTON SCTTERING AND THE PSEUDOSCALAR MESON THEORY

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ABSTRACT The paper embodies a study of the p-p scattering in the light of pseudoscalar meson theory using Born approximation. The theoretical findings have been numerically tested for energies of 150, 240, 340 Mev. The analysis indicates that either the pseudoscalar theory or a combination of the central and tensor force term of the interaction is unable to explain the isotropy in scattering observed experimentally.

INTRODUCTION

With the discovery and operation of different types of high energy particle accelerators, a considerable amount of data on the differential cross section for p-p scattering are now available. The energy region for p-p scattering ranges from 0 to 340 Mev. These experiments show that the p-p scattering is nearly isotropic except at small angles where deviations from isotropy occur for small energy particles due to relative importance of the Coulomb field. It is our object here to see how best we can fit the experimental results with the theoretical findings based on interactions derived from the field-theories. The investigation is based upon the assumption that the interaction between two nucleons may be represented by means of a static potential that may have a charge and spin dependence. The static potential signifies that the terms in the potential expression which depend on nucleon velocities are neglected in the expression for the interaction considered.

The 340 Mev scattering experiments indicate a nearly spherically symmetric distribution over a wide angular range (15° to 90°) in the centre of mass system with a value close to $4.5 \times 10^{-27}\text{cm}^2/$ steradian. The experimental values suggest that the process bears a similarity to scattering of plane waves by a perfectly rigid sphere which represents a short range potential of range equal to its radius. To explain isotropy with such a model the wavelength of the proton should be large compared to the range. The numerical value of the range which would give an isotropic differential cross section of magnitude $4.5 \times 10^{-27}\text{cm}^2/$ steradian is approximately 1×10^{-13}cm. This is greater than the wavelength of the proton at this energy and is approximately double of it. So it is not possible to explain isotropy with such a model. The experiments also indicate that the differential cross section exhibits isotropy at all energies down to 100 Mev with a little increase in the absolute magnitude. An attempt to explain the phenomena with a central force model is unsuitable because it gives a strong forward scattering. The results obtained do not exhibit isotropy in the differential cross section even over a small angular range at high or low energies. The presence of electrical quadrupole moment of deuteron suggests that the nuclear forces are non-central in character. Hence the potential energy of a pair of nucleons depends not only on their distance of separation but also on their relative orientation with respect to the spin axis. This non-central or tensor force might compensate the magnitude of the differential cross sections obtained from central forces in such a way that the resultant effect exhibits isotropy both at high and low energies. Rarita and Schwinger (1941) first considered the effect of the tensor force with n- p scattering assuming that the radial dependence of both the central and non-central forces is the same. The results of their investigation explain the quadrupole moment of deuteron but cannot explain the scattering results at 90 Mev.

TENSOR FORCE

To get an idea about the effect of tensor force on the p-p differential cross section, we discuss the same below using Born approximation and a potential of the from :

$$S_{12}.\frac{f^2}{4\pi}e^{-\chi r} \tag{1}$$

where S_{12} = the tensor force operator

$$= \underline{3(\boldsymbol{\sigma}_1\mathbf{r})}(\boldsymbol{\sigma}_2\,\mathbf{r}) - (\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2) \tag{2}$$

Let the momentum of the incident porton in the centre of mass system be $k\hbar\,\mathbf{n}_0$ and let $k\hbar\,\mathbf{n}$ be its mementum after scattering. The vectors $\mathbf{n}_0$ and $\mathbf{n}$ (figure 1) represent the unit vectors in the directions of initial relative motion and scattering respectively. We would consider $\mathbf{n}_0$ also as the axis of quantisation of the relevant spin-functions.

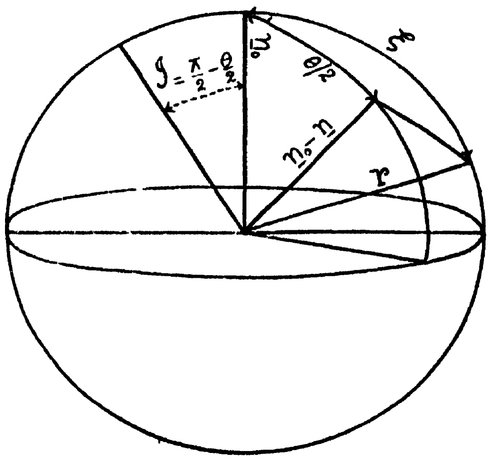

FIG. 1

With the above tensor force the amplitude of scattering according to Born's approximation takes the following form :

$$F(\theta) = -\frac{M}{4\pi\hbar^2}\int e^{i(\mathbf{n}_0-\mathbf{n})Kr}\,\chi_{ms'}\left\{\frac{3(\boldsymbol{\sigma}_1\mathbf{r})(\boldsymbol{\sigma}_2\mathbf{r})}{\mathbf{r}^2} - \boldsymbol{\sigma}_1\boldsymbol{\sigma}_2\right\}\chi_{ms}\,.\,\frac{f^2\,e^{-\chi r}}{4\pi r}\,d\tau \quad \ldots \quad (3)$$

The tensor force operator for the proton-proton system when applied to any singlet state gives zero as its eigen value. So we consider here its effect on triplet scattering only. If the spins of the two protons are parallel the system is said to be in the triplet state and if the spins are antiparallel the system is said to be in the singlet. The spin dependent parts of the wave functions are :

$$^1\chi_0 = \frac{1}{\sqrt{2}}\left\{\alpha(1)\beta(2) - \alpha(2)\beta(1)\right\}(\text{singlet})$$

$$\left.\begin{aligned} ^3\chi_1 &= \alpha(1)\alpha(2) \\ ^3\chi_0 &= \frac{1}{\sqrt{2}}\left\{\alpha(1)\beta(2) + \alpha(2)\beta(1)\right\} \ (\text{triplet}) \\ ^3\chi_{-1} &= \beta(1)\beta(2) \end{aligned}\right. \quad \ldots \quad (4)$$

The symbols (1) and (2) of the right hand side of equation (4) refer to the spin coordinates of proton number one and two respectively. The numbers 1, 0, -1 on the left hand side respresent the magnetic quantum numbers. α and β are the ordinary spin functions of a nucleon. The result of operating the tensor force operator S_{12} on the triplet spin functions for various values of magnetic quantum numbers $(1, 0, -1)$ yields the following matrix :

ms / ms'	1	0	-1
1	$(3\cos^2\xi - 1)$	$3\sqrt{2}\cos\xi\sin\xi e^{-i\eta}$	$3e^{-2i\eta}\sin^2\xi$
$(A_{ms\,ms'}) = 0$	$3\sqrt{2}\sin\xi\cos\xi e^{i\eta}$	$-2(3\cos^2\xi - 1)$	$-3\sqrt{2}\cos\xi\sin\xi e^{-i\eta}$
-1	$3\sin^2\xi e^{2i\eta}$	$-3\sqrt{2}\sin\xi\cos\xi e^{i\eta}$	$(3\cos^2\xi - 1)$

Substituting the elements of the above matrix in (3) and performing the integration in each case after transformation of coordinates from r, ξ, η to r, θ, ϕ as indicated in the diagram we would obtain the values of $F(\theta)$ corresponding to each element of the matrix. For the case ms, $ms' = 1$, 1 ; we get :

$$F(\theta) = -\frac{Mf^2}{4\pi\hbar^2}\int e^{iKr\cos\theta'}\,.\,2\,\frac{(3\cos^2\xi - 1)}{2}\,.\,\frac{e^{-\chi r}}{4\pi r}\,.\,2\pi r^2 dr.d(\cos\theta')$$

$$= -\frac{Mf^2}{4\pi\hbar^2}\,.\,\frac{(3\sin^2\theta/2 - 1)}{2}\,.\,2\int_0^\infty\left\{\frac{3(\sin Kr - Kr\cos Kr)}{K^3r^3} - \frac{\sin Kr}{Kr}\right\}\times \frac{e^{-\chi r}}{r}\,.\,r^2 dr$$

$$= -\frac{Mf^2}{4\pi\hbar^2}\,.\,\frac{(3\sin^2\theta/2 - 1)}{2}\,.\,2\left[\frac{3}{K^2} - \frac{3\chi}{K^3}\tan^{-1}\frac{K}{\chi} - \frac{1}{K^2+\chi^2}\right] \quad \ldots \quad (5)$$

where $K = 2k\sin\theta/2$

Obtaining thus the values of $F(\theta)$ for other elements of the matrix an average of their squares over the various spin directions is determined. The sum of each averages would represent the intensity of scattering in the triplet state. Performing as stated above, we have,

$$\underset{\text{triplet}}{I(\theta)} = 8\left(\frac{Mf^2}{4\pi\hbar^2}\right)^2\left(\frac{3\sin^2\theta/2-1}{2}\right)^2\left(\frac{3}{K^2}-\frac{3X}{K^3}\tan^{-1}\frac{K}{X}-\frac{1}{K^2+X^2}\right)^2 \quad \ldots (6)$$

It is clear from (6) that the intensity of the triplet scattering does not depend on ϕ as during integration and averaging such terms vanish. This is evident because the scattering of an unpolarised beam of protons is axially symmetric.

For an unpolarised beam of protons, we have :

$$I(\theta) = \tfrac{1}{4}\underset{\text{singlet}}{I(\theta)} + \tfrac{3}{4}\underset{\text{triplet}}{I(\theta)}$$

$$= \tfrac{3}{4}\underset{\text{tiplet}}{I(\theta)} \qquad (\because \underset{\text{singlet}}{I(\theta)} = 0 \text{ in the case considered})$$

$$\tfrac{3}{4}.8.\left(\frac{Mf^2}{4\pi\hbar^2}\right)^2.(B_{ms\,ms'})^2.\ C^2(\theta)$$

$$= 6\left(\frac{Mf^2}{4\pi\hbar^2}\right)^2.(B_{ms\,ms'})^2.\ C_2(\theta) \qquad (7)$$

where $$C^2(\theta) = \frac{3}{K^2} - \frac{3X}{K^3}\tan^{-1}\frac{K}{X} - \frac{1}{K^2+X^2}. \qquad (8)$$

and $B_{ms\,ms'}$ represents the diagonal elements of the matrix given below. The matrix here is derived from the matrix $(A_{ms\,ms'})$ after transformation of cordinates as stated before.

$ms \backslash ms'$	1	0	−1
1	$(\sin^2\theta/2-1)$	$3\sqrt{2}\sin\theta/2\cos\theta/2e^{-i\varphi}3$	$\cos^2\theta/2e^{-2i\varphi}$
0	$3\sqrt{2}\sin\theta/2\cos\theta/2e^{i\varphi}$	$-2(3\sin^2\theta/2-1)$	$-3\sqrt{2}\sin\theta/2\cos\theta/2e^{i\varphi}$
−1	$5\cos^2\theta/2e^{2i\varphi}$	$-3\sqrt{2}\sin\theta/2\cos\theta/2e^{i\varphi}$	$(3\sin^2\theta/2-1)$

The computed values of the differential cross sections at 240 Mev for various centre of mass angles are in Table II and represented in curve I(b) in figure 2. For comparison the corresponding values for simple Yukawa potential are given in Table I and represented in curve I(a).

The curves I(c), (d) and (e) represent the experimental values at 105, 240 and 340 Mev.

TABLE I

Centre of mass angles in degrees	30°	40°	50°	60°	70°	80°	90°
$\sigma(\theta)$ in mb/std	3.96	1.78	.88	.48	29	.25	.18

TABLE II

Angles in degrees c. m. system	30°	40°	50°	60°	70°	80°	90°
$\sigma(\theta)$ mb/std.	.68	.44	.27	211	211	.197	.204

In these numerical computations we have taken the value of $\frac{f^2}{\hbar c} = .7$ for the symmetrical case as has been obtained after normalising the value of the total n-p scattering cross section at 90 Mev determined experimentally by Fox, Leith, Wouters and Mackenzie (1950) and Dejuren (1950).

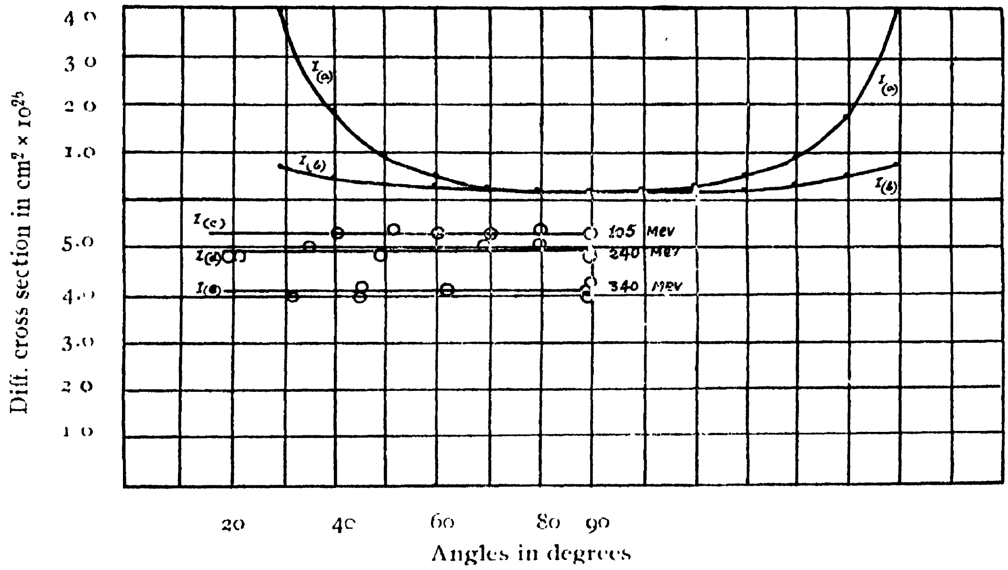

FIG. 2

Curves 1(a) and 1(b) represent the variation of σ (θ) with angles at 240 Mev for Yukawa potential and for tensor force without singularity respectively. Curves 1 (c), (d), and (e) represent experimental curves.

With central forces above the curve shows a tendency to drop down at 90°. The extent of dropping at 240 Mev is, however, not much perceptible. The tensor above gives a peak at 90° which also is not very marked at this energy. It is therefore expected that a combination of the two for triplet scattering would flatten the curve at 90°. Ashkin and Wu have, however, shown that the peak at 90° turns to a trough at 90° with increase of energy. If the symmetrical forces are considered the trough or the peak becomes so small that the curve at 90° is practically flat. The magnitude of the differential cross section drops to a very small value if the tensor force alone is taken into consideration. The curve, however, remains fairly flat over the angular range 50° to 90°. A combination of the central and tensor forces, however, increases the magnitude of the differential cross section at small angles but at large angles the values remain still smaller. In case of symmetrical forces also the magnitude of the differential cross sections is too low particularly at large angles though the curve is sufficiently flat as in the case of tensor forces. If the symmetrical forces are not considered the curve is no longer flat and though there is an increase in the magnitude of differential cross section at small angles (30°) but at large angles it drops to a very small value as has been observed in the case of simple Yukawa potential. It is apparent, therefore, that such combinations of central and non-central forces will not lead to the results that would exhibit isotropy over the angular range in which the experiments show isotropic scattering and at the same time would increase the magnitude of the differential cross section even approximately to the experimental value.

PSEUDOSCALAR INTERACTION

Meson field theories based on the assumption of weak coupling between nucleon and meson fields suggest that the main contribution to the interaction energy arises from the static part of the meson field. Each one of the possible types of meson fields gives rise to a form of static interaction between a pair of nucleons.

Recent experiments on meson-nucleon interaction favours the pseudoscalar nature of the meson. On decay a π° meson gives two γ-rays. So its spin cannot be unity. The reactions which arise in the capture of π^- meson by deuteron indicate that it is not scalar. An analysis of the cross sections of production (p-p collision) and of absorption (in deuteron) of π^+ meson indicates that its spin value should be zero. The fact that the cross section for production of π^+ meson is very large compared to that of π° meson, suggests that the π° meson is pseudoscalar in nature. Since it is quite natural to assume the same nature for all the three kinds of mesons for symmetry, the π meson, supposed to be responsible for nuclear forces, should preferably be regarded pseudoscalar in nature.

We investigate here, using non-relativistic Born approximation, the extent to which the differential cross sections as calculated from the static potential of the interaction suggested by the symmetrical pseudoscalar meson theory agrees with the experiments.

The static potential obtained from the symmetrical pseudoscalar theory is given by

$$V_{ps} = \tfrac{1}{3}(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)\left[(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2) + \left\{\frac{3(\boldsymbol{\sigma}_1 \mathbf{r})(\boldsymbol{\sigma}_2\mathbf{r})}{r^2} - (\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)\right\}\left\{1 + \frac{3}{\chi r} + \frac{3}{\chi^2 r^2}\right\}\right]\frac{f^2}{4\pi}\cdot\frac{e^{-\chi r}}{r} \quad \ldots \quad (9)$$

The amplitude scattered, using Born-appx. is,

$$F(\theta) = -\frac{Mf^2}{4\pi h^2}\cdot\tfrac{1}{3}(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)\cdot\frac{1}{4\pi}\left[\int_0^\infty\int_{-1}^{+1} e^{iKr\cos\theta'}\chi_{ms'}\left\{(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2) + S_{12}\left(1 + \frac{3}{\chi r} + \frac{3}{\chi^2 r^2}\right)\right\}\right.$$
$$\left.\times\chi_{ms}\cdot\frac{e^{-\chi r}}{r}\cdot dt\right] \quad \ldots \quad (10)$$

For singlet states $\chi_{m_s'}(\sigma_1\sigma_2)\chi_{m_s} = -3$ and for triplet states it is equal to unity. The effect of the tensor force operator on singlet and triplet states has been discussed before. Similar calculations yield,

$$F(\theta) = -\frac{M}{4\pi h^2}\cdot\frac{f^2}{4\pi}\cdot\frac{1}{3}(\tau_1\tau_2)\Bigg[\begin{pmatrix}-3\\ 1\end{pmatrix}\int_0^\infty\int_{-1}^{+1} e^{iKr\cos\theta'}\cdot\frac{e^{-\chi r}}{r}\,d\tau + \begin{pmatrix}0\\ A_{m_s m_s'}\end{pmatrix}\int_0^\infty\int_{-1}^{+1} e^{iKr\cos\theta'}\frac{e^{-\chi r}}{r}\,d\tau\Bigg]$$

$$= -\frac{M}{4\pi h^2}\cdot\frac{f^2}{4\pi}\cdot\frac{(\tau_1\tau_2)}{3}\cdot 4\pi\Bigg[\begin{pmatrix}-3\\ +1\end{pmatrix}\int_0^\infty\frac{\sin K_r}{K_r}\cdot\frac{e^{-\chi r}}{r}\cdot r^2 dr - \begin{pmatrix}0\\ B_{m_s m_s'}\end{pmatrix}$$

$$2\int_0^\infty\left\{\frac{3(\sin K_r - K_r\cos K_r)}{K^3r^3} - \frac{\sin K_r}{K_r}\right\}\times\left(1+\frac{3}{\chi r}+\frac{3}{\chi^2r^2}\right)r^2 dr$$

$$= -\frac{Mf^2}{4\pi h^2}\cdot\frac{(\tau_1\tau_2)}{3}\left[\begin{pmatrix}-3\\ +1\end{pmatrix}\frac{1}{K^2+\chi^2} - 2\begin{pmatrix}0\\ B_{m_s m_s'}\end{pmatrix}\frac{K^2}{\chi^2}\cdot\frac{1}{K^2+\chi^2}\right] \quad \ldots \quad (11)$$

$A_{m_s m_s'}$ and $B_{m_s m_s'}$ represent the elements of the matrices $(A_{m_s m_s'})$ and $(B_{m_s m_s'})$. In the case of p-p interaction, $(\tau_1\tau_2) = 1$, since the system is always charge symmetric.

$$\therefore\quad \underset{\text{singlet}}{I(\theta)} = \left(\frac{Mf^2}{4\pi h^2}\right)^2\cdot\left(\frac{1}{K^2+\chi^2}\right)^2$$

$$= \left(\frac{Mf^2}{4\pi h^2}\right)\cdot f^2(\theta). \quad \ldots \quad (12)$$

$$\text{and}\quad \underset{\text{triplet}}{I(\theta)} = \left(\frac{M}{4\pi h^2}\right)^2\frac{f^4}{9}\left\{f^2(\theta) + 8\frac{(3\sin^2\theta/2 - 1)^2}{4}\cdot\left(\frac{K^2}{\chi^2}\cdot\frac{1}{K^2+\chi^2}\right)^2\right\}$$

$$= \left(\frac{M}{4\pi h^2}\right)^2\cdot\frac{f^4}{9}\cdot\left[f^2(\theta) + 8C^2(\theta)\right] \quad \ldots \quad (13)$$

where $f(\theta) = \frac{1}{K^2+\chi^2}$ and $C^2(\theta) = \frac{K^2}{\chi^2}\,\frac{1}{K^2+\chi^2}\,P_2(\sin\theta/2)$

For the symmetrical case the expressions for the singlet and triplet differential cross sections are,

$$\underset{\text{singlet}}{I(\theta)} = \left(\frac{M}{4\pi h^2}\right)^2\cdot f^4(f(\theta) + f(\pi-\theta))^2 \quad \ldots \quad (14\ a)$$

$$\underset{\text{triplet}}{I(\theta)} = \left(\frac{M}{4\pi h^2}\right)^2\cdot\frac{f^4}{9}\left[\left\{f(\theta) - f(\pi-\theta)\right\}^2 + 8\left\{C^2(\theta) + C^2(\pi-\theta) + C(\theta)\,C(\pi-\theta)\right\}\right] \quad \ldots \quad (14\ b)$$

Hence the differential cross section for an unpolarised beam of protons is,

$$I(\theta) = \tfrac{1}{4}\, I_{\text{sing}}(\theta) + \tfrac{3}{4}\, I_{\text{triplet}}(\theta)$$

$$= \left(\frac{Mf^2}{4\pi h^2}\right)^2 \cdot \left[\frac{1}{4}\Big(f(\theta) + f(\pi-\theta) \Big)^2 + \frac{1}{9}\cdot\frac{3}{4}\Big(f(\theta) - f(\pi-\theta) \Big)^2 + \frac{1}{9}\cdot\frac{3}{4}\cdot 8\Big(C^2(\theta) + C^2(\pi-\theta) + C(\theta)C(\pi-\theta) \Big) \right]$$

$$= \left(\frac{Mf^2}{4\pi h^2}\right)^2 \cdot \left[\frac{1}{4}\Big(f(\theta) + f(\pi-\theta) \Big)^2 + \frac{1}{12}\Big(f(\theta) - f(\pi-\theta) \Big)^2 + \frac{2}{3}\Big(C^2(\theta) + C^2(\pi-\theta) + C(\theta)C(\pi-\theta) \Big) \right] \quad \ldots \quad (15)$$

The computed values of the differental cross sections from the equations (11) to (15) above for energies 150 Mev, 240 Mev and 340 Mev for various angles in the centre of mass system are given in Table III and the relevant curves are shown in II (*a*) (*b*) (*c*) in figure 3. A separate study of the effect of the factor $\left(\frac{1}{3} + \frac{1}{\chi} + \frac{1}{\chi^2 r}\right)$ occuring along with the tensor force operator appearing in the pseudoscalar expression for potential, has also been made. Table IV gives the values of the differential cross sections for this part of the potential only. The relevant curves are represented in curves III (*a*), (*b*) and (*c*) in figure 4. The procedure is quite justified since the central and tensor forces contribute separately to the cross section.

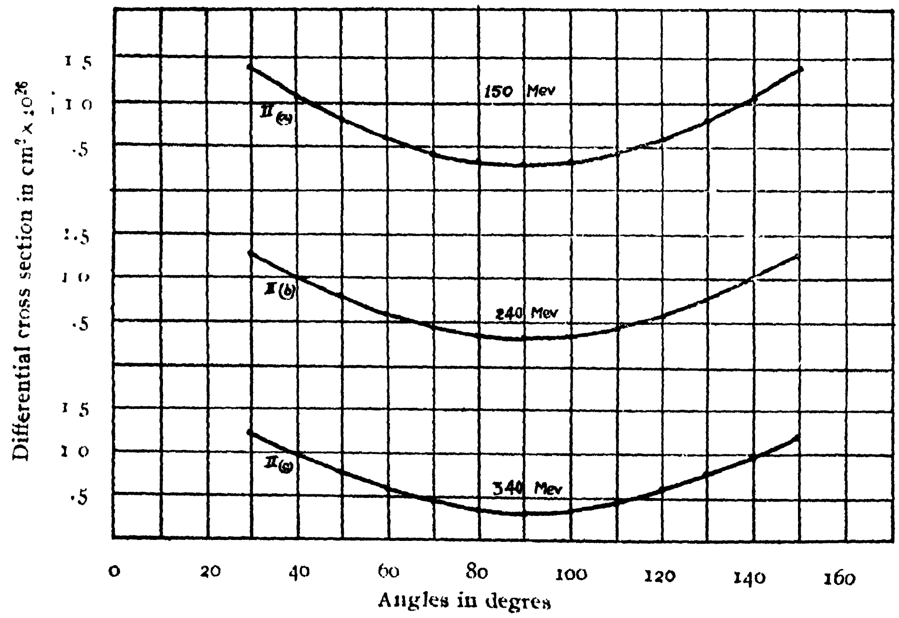

FIG. 3

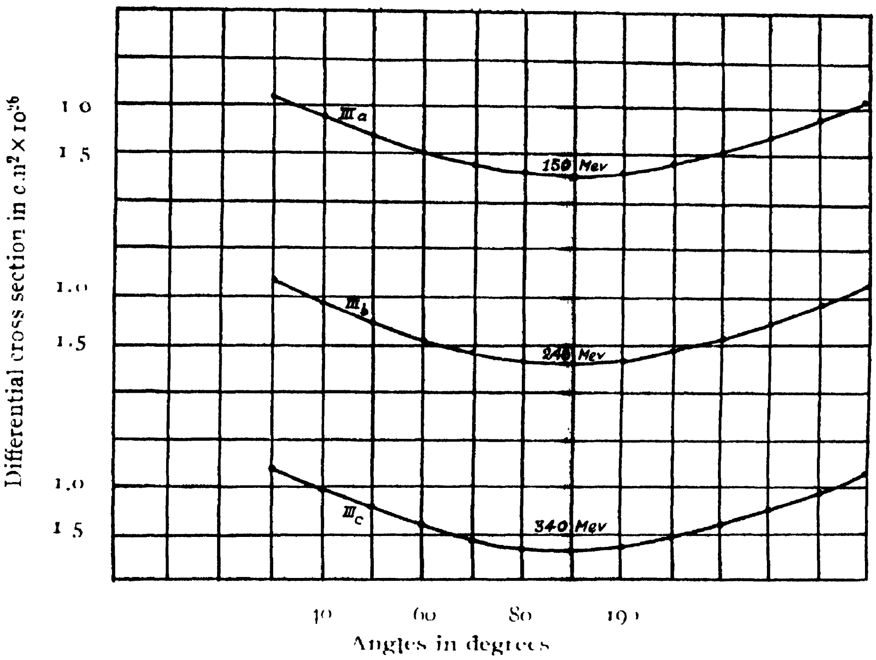

FIG. 4

TABLE III

Angles in degrees, c. m. system		30°	40°	50°	60°	70°	80°	90°
σ (θ) mb/std	150 Mev	13.71	10.52	8.02	5.99	4.46	3.53	3.16
	240 Mev	12.81	9.93	7.93	5.88	1.43	3.56	3.20
	340 Mev	12.37	9.78	7.83	5.91	4.54	3.60	3.21

TABLE IV

Angles in degrees, c. m system		30°	40°	50°	60°	70°	80°	90°
σ (θ) mb/std	150 Mev	10.80	8.90	7.02	5.30	3.94	3.08	2.74
	240 Mev	11.16	9.03	7.64	5.57	4.21	3.37	3.02
	340 Mev	11.31	9.28	7.56	5.77	4.42	3.50	3.11

CONCLUSION

The above analysis shows that the differential cross section at small angles is larger in magnitude than that at large angles. In the angular range 30° to 60°, the magnitude of the differential cross section decreases quite rapidly, whereas, in the interval 60° to 90°, the rate of decrease is comparatively small. Moreover, the decrease of the value of the differential cross section with angle becomes smaller as the energy of the particles becomes larger, this shows that the curves show a tendency to flatten as the energy increases. A comparison of Tables III and IV shows that the contribution to the value of the differential cross section comes mostly from the tensor force term which has $1/r^3$ singularity ; it is worth noting that the scattering cross section due to the tensor force with the $1/r^3$ singularity increases with increasing energy, whereas, the scarttering cross section due to the tensor force without the $1/r^3$ singularity term decreases with energy (*cf.* Ashkin and Wu 1948). This means that for the pseudoscalar interaction the increase of the scattering cross section with increasing energy is entirely due to the $1/r^3$ singulary term of the interaction.

The pseudoscalar interaction consists of a central force and a tensor force both of which give decreasing values of the scattering cross section with increasing angle, that being so it is not possible to explain the isotropy of scattering with different combinations of these two forces alone.

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